

Group Means and Elemental Means.—

Given a set of elements X partitioned into N subsets X_i , we define n_i as the number of elements in the i th subset. If each subset is assigned a value for a secondary variable r_i , and this variable r is positively correlated with n_i , then by definition

$$\sum_i (r_i - \bar{r})(n_i - \bar{n}) > 0, \quad (1)$$

or equivalently

$$\sum_i n_i r_i - N \bar{r} \bar{n} > 0. \quad (2)$$

This may be rearranged as follows

$$\frac{\sum_i n_i r_i}{\sum_i \bar{n}} > \bar{r}, \quad (3)$$

where the expression on the left hand side is the definition of the elemental mean \bar{r}_e . The elemental mean therefore always exceeds the group mean

$$\bar{r}_e > \bar{r}. \quad (4)$$